Quantification of frequency-dependent absorption phenomena

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Abstract
Of all fluid and solid properties, quantities that describe losses are among the most challenging to quantify. In part, this is due to superimposed dissipative mechanisms, such as diffraction effects from spatially limited sources. Inherent to all these phenomena, however, is a specific frequency dependence. The nature of the frequency dependence varies, resulting from the respective absorption mechanism. Pure fluids, for example, exhibit absorption of acoustic waves with quadratic frequency dependence[1]. In solids, there are several absorption models that can be applied, each having different characteristics with respect to frequency. Other dissipative effects, such as diffraction, also show frequency dependence. In an approach using the raw moments of the signals from acoustic transmission measurements, a method to quantify absorption and dissipation phenomena with arbitrary frequency dependence is presented. The described method is applied to different absorption measurement problems. To verify that accurate results can be achieved under ideal conditions, the method is applied to signals generated using acoustic field simulation with different absorption models. To show its numerical stability, it is used qualitatively to evaluate the absorption of a fluid at different thermodynamic states.

Keywords
Absorption, frequency dependence, dissipation, attenuation

I. Motivation

While measurements of the acoustic absorption properties of fluids and solids can be used for an in-depth modelling of the acoustic behaviour of a material, there are other applications as well. In pure fluids, for example, acoustic absorption is described by three separate physical quantities[2] (shear viscosity, volume viscosity and thermal conductivity) and can thus be used to measure these fluid properties. For each absorption process, different frequency dependencies are either assumed in the modelling process based on observations, or result from the underlying physical properties of the respective wave propagation. In addition to absorption caused by the fluid, dissipative effects may occur in a measurement system, such as e.g. microstructural sound scattering and diffraction effects from a spatially limited acoustic transducer. As these effects also generally show frequency dependence, they can be quantified using the presented method as well.

II. Methodology

It is assumed that an ultrasonic transducer emits an acoustic wave with magnitude spectrum $|U_0(\mathrm{j}\omega)| = U_0(\omega)$. Assuming that the sound propagation is linear and the superposition principle applies, the change to the spectrum caused by frequency-dependent absorption $\alpha(\omega)$ can be modelled using an exponential decay function:

$$U(\omega, z) = U_0(\omega)\mathrm{e}^{-\alpha(\omega)z}\,, \tag{1}$$

where z is the propagation distance of the acoustic wave. Similar to approaches that apply the change in centre frequency of the acoustic signal to determine absorption[3, 4, 5], the i-th raw moment of the spectrum $U(\omega, z)$ is considered:

$$m_{i,U}(z) = \int_{-\infty}^{\infty} \omega^i U(\omega, z)\mathrm{d}\omega\,. \tag{2}$$

For the following steps, the frequency-dependent absorption is assumed to be in a general polynomial form with the polynomial absorption parameters a_n:

$$\alpha(\omega) = \sum_n a_n \omega^n . \tag{3}$$

To analyse how the absorption process influences the raw moments of the acoustic signal spectrum as the acoustic wave propagates, the derivative ∂_z of the raw moments (equation 2) with respect to z is analysed. Inserting equations 1 and 3 yields:

$$\partial_z m_{i,U}(z) = \int_{-\infty}^{\infty} \omega^i \left(-\sum_n a_n\, \omega^n\right) U_0(\omega) \exp\left(-z \sum_n a_n \omega^n\right) \mathrm{d}\omega\,, \tag{4}$$

which can again be written using raw moments:

$$\partial_z m_{i,U}(z) = -\sum_n a_n \int_{-\infty}^{\infty} \omega^{i+n} U_0(\omega) \exp\left(-z \sum_n a_n \omega^n\right) \mathrm{d}\omega = -\sum_n a_n\, m_{i+n,U}(z)\,. \tag{5}$$

Using this relationship, different methods to determine the parameters a_n of the absorption can be derived. Here, two approaches are presented: For measurement systems where the acoustic signal can only be recorded at two distances z_1 and z_2, a procedure to quantify an absorption mechanism with one polynomial parameter is shown. Further, for measurement systems that acquire signals at multiple distances z, a method to determine dissipative phenomena with multiple polynomial parameters is presented.

II.1. Single parameter absorption models

For single parameter absorption mechanisms, as they occur for example with quadratic frequency dependence ($n = 2$) in pure fluids, equation 5 can be simplified as follows:

$$\partial_z m_{i,U}(z) = -a_2\, m_{i+2,U}(z)\,. \tag{6}$$

While a raw moment of arbitrary order of the magnitude spectrum of an acoustic signal can be calculated easily via Fourier transform, the derivative has to be estimated. This is especially true if signals at only two distances are recorded. The straightforward method of estimating $\partial_z m_{i,U}(z)$ would be to use a difference quotient, however, this would only yield a good estimate if $m_{i,U}(z)$ is linear in z. Evaluating simulation results and theoretical considerations for Gaussian shaped spectra however show, that $m_{i,U}(z)$ decays exponentially if bandwidth is small. Hence, $m_{i,U}(z)$ can be modelled using a simple exponential expression ($m\mathrm{e}^{-nz}$) which can then be derived analytically. The derivative of the raw moment can be inserted in equation 6 and yields the following expression, assuming $i = 0$ to minimize numerical errors:

$$a_2 = \frac{1}{z_2 - z_1} \frac{m_{0,U}(z_1)}{m_{2,U}(z_1)} \ln\left(\frac{m_{0,U}(z_1)}{m_{0,U}(z_2)}\right). \tag{7}$$

Equation 7 bears some resemblance to the expression attained by assuming monochromatic plane waves with absorption, evaluating the amplitude and solving for the absorption α. However, it directly yields the frequency independent parameter a_2 and thereby solves the problem that occurs in real measurement systems, where the waves are not monochromatic. Equation 7 can further be modified for other single-parameter absorption models by changing the order of the raw moment $m_{2,U}(z_1)$ to match the power of the absorption mechanism to be quantified.

II.2. Multi parameter absorption models

For measurement systems that acquire signals at multiple distances, one can take advantage of the fact that equation 5 needs to hold for all distances z. This leads to a system of equations, which can be solved for the absorption parameters a_n. As an example, the equation system to be solved if the signal is acquired in three positions z_1, z_2 and z_3, and a constant and a linear absorption parameter a_0 and a_1 are to be estimated, takes the following form:

$$-\begin{pmatrix} \partial_z m_{0,U}(z_1) \\ \partial_z m_{0,U}(z_2) \\ \partial_z m_{0,U}(z_3) \end{pmatrix} = \begin{pmatrix} m_{0,U}(z_1) & m_{1,U}(z_1) \\ m_{0,U}(z_2) & m_{1,U}(z_2) \\ m_{0,U}(z_3) & m_{1,U}(z_3) \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}, \tag{8}$$

again assuming $i = 0$. While this example is already overdetermined, it proves helpful for an application of this method that the signal can be acquired at a significantly higher number of positions than the number of absorption parameters to be determined. In this case, due to the high number of positions, the derivative of moment of $\partial_z m_{0,U}(z)$ can be determined by difference quotients.

III. Evaluation

The performance of the presented approaches is evaluated by applying them to different signals from simulations and measurements:
To evaluate whether exact results can be expected under ideal conditions, virtual measurement signals are generated using a one-dimensional representation of a precision sound velocity measurement setup (figure 1), filled with a fluid medium (sound velocity $c =$ 1500 m/s, density $\rho_0 =$ 1000 kg/m³). To cause absorption, the medium is provided with a volume viscosity μ_v of 5 mPa s. With no other dissipative mechanisms active, this causes a rather small quadratic absorption parameter $a_2 = \mu_\mathrm{v}/(2\,c^3\rho_0)$[2] of $7.4074{\cdot}10^{-16}$ s²/m. Simulations are conducted using finite differences in time domain (FTDT). The transmitted signal is a Gaussian modulated sinusoidal pulse with a centre frequency of 8 MHz and a relative bandwidth of 0.1. Evaluating equation 7 for the signals generated using the simulation procedure yields an estimated volume viscosity of 4.966 mPa s. Even though the absorption is small, the relative deviation from the expected result is only 0.68 %. This deviation is likely to be caused by numerical effects in the simulation.

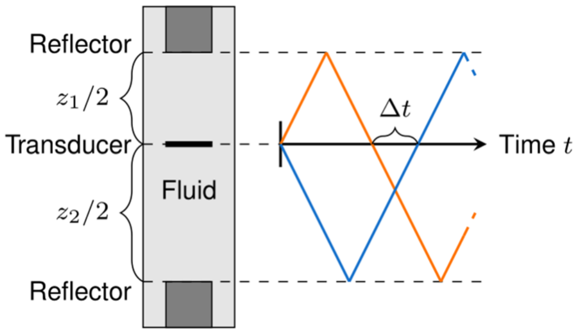

Figure 1: Schematic of the precision sound velocity measurement setup[6] with two reflectors and the propagation of acoustic signals over time.

To evaluate the performance of the algorithm when applied to measurement data, it is used to estimate the acoustic loss of methanol in different thermodynamic states from signals recorded using the aforementioned measurement setup originally designed for precision sound velocity measurement[6] (figure 1). The physical setup consists of a single, suspended quartz disc transducer to allow for sound emission in both directions and two acoustic reflectors mounted at different distances from the transducer. Due to the differently positioned reflectors, the acoustic bursts that are emitted both propagate over different distances until they reach the transducer again, yielding the two signals necessary for the application of equation 7. While the resulting numerical values (figure 2) are still superimposed by the influence of the measurement setup, such as diffraction, they show the numerical stability of the approach, as similar thermodynamic states of the fluid result in similar (classical) acoustic loss $\mu_\mathrm{meas} = 2a_{2,\mathrm{meas}} \cdot \rho_0 c^3$. Especially in comparison to the evaluation of the change in centre frequency discussed before[5], the presented method yields stable results even though the properties of the transducer, and thus those of the transmitted signal, change with temperature.
The estimation method for multi parameter absorption models is evaluated by applying it to virtual measurement results generated by performing a finite element simulation of a circular waveguide with a radius of 1 mm using CFS++[7]. The waveguide is filled with the same fluid

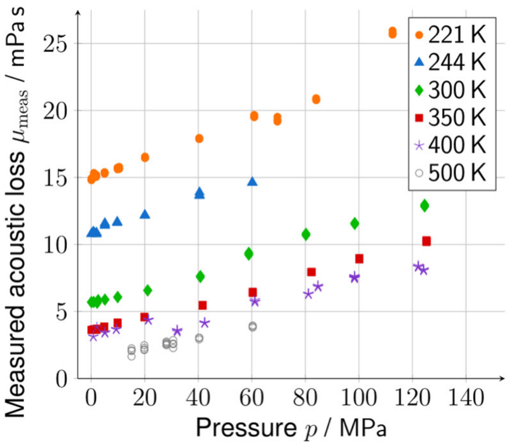

Figure 2: Estimated acoustic loss of methanol at different temperatures and pressures.

as used in the preceding simulation. However, losses are now caused by Rayleigh absorption[8] with parameters $\alpha_{\mathrm{M}} = 10^3\,\mathrm{s}^{-1}$ and $\alpha_{\mathrm{K}} = 10^{-9}\,\mathrm{s}$ as an example for an absorption mechanism with multiple parameters. The waveguide is excited with a sinusoidal burst at 2 MHz while 160 signals are recorded at different distances along the waveguide with a pitch of 0.3 mm. Applying equation 8 to these signals leads to a highly overdetermined equation system that can be solved using a least squares approach. To directly compare the results, the relation between the Rayleigh absorption parameters and the absorption α has to be established, leading to a constant and a quadratic absorption parameter a_0 and a_2. The resulting estimate for $\alpha_{\mathrm{K}} = 1.002 \cdot 10^{-9}\,\mathrm{s}$ deviates only slightly from the expected result. The estimate for $\alpha_{\mathrm{M}} = 1.9 \cdot 10^3\,\mathrm{s}^{-1}$, however, shows significant deviation, probably due to the relatively low absorption caused by this parameter in the examined frequency range in comparison to α_{K}.

IV. Conclusion

The presented method to determine the parameters of frequency-dependent absorption and dissipative mechanisms shows promising results in delivering numerically stable estimates for attenuation parameters, even if the properties of the transducer change and the signal cannot be assumed to be monochromatic. Prerequisites for the quantification of absorption models with multiple parameters using the presented method are that all parameters cause significant absorption in the analysed signal and that signals are recorded at multiple distances from the acoustic source. As the presented procedures only rely on the evaluation of the spectrum of the signals, it is possible to apply them in other fields, for example optics, as well.

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