# RESEARCH ARTICLE 

# On the difference between a point multipole and an equivalent <br> linear arrangement of point charges in force field models for vapour-liquid equilibria. Partial charge based models for 59 real fluids 

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#### Abstract

The replacement of a point dipole and a point quadrupole by a corresponding linear arrangement of two point charges $(+q,-q)$ and accordingly three point charges $(+q,-2 q,+q)$ is studied with respect to vapour-liquid equilibria. The dependence of saturated liquid density, vapour pressure and heat of vaporisation on the choice of the distance $d$ between the charges in the point charge arrangement is analysed. For the studied dipolar two-centre LennardJones (2CLJD) and quadrupolar two-centre Lennard-Jones (2CLJQ) models, $d / \sigma$ between $1 / 15$ and $1 / 20$ is a reasonable compromise between numerical and physical accuracy, where $\sigma$ is the Lennard-Jones size parameter. The results are used to derive validated partial charge based models of 59 real fluids from previously published point dipole and point quadrupole based models.


Keywords: molecular modelling; polar interactions; vapour-liquid equilibrium

## 1. Introduction

Lennard-Jones (LJ) based molecular models with superimposed electrostatic sites are a widely used class of force fields for molecular simulations of thermophysical properties of fluids. Stoll et al. [1] and Vrabec et al. [2] have modelled 38 real fluids with rigid two-centre LJ (2CLJ) models with a superimposed point dipole and 21 fluids with a superimposed point quadrupole. The four model parameters (LJ size $\sigma$ and energy $\epsilon$, elongation $L$, i.e. distance between the LJ sites, and dipole moment $\mu$ or quadrupole moment $Q$ ) were determined from a fit to correlations of experimental vapour-liquid equilibrium (VLE) data. Polar 2CLJ models strongly simplify the intermolecular interactions, e.g. the asymmetry of the molecules is neglected and the polar interaction is always aligned along the main molecular axis. Also the polarizability, which is often assumed to be a crucial molecular property for thermodynamics, is only implicitly considered by LJ interaction sites. Furthermore, the internal degrees of freedom are neglected as the polar 2CLJ models are rigid. However, it has been shown in many applications [3-7] that these models have an excellent predictive power for various properties, including transport data.

[^0]Therefore, it is not in the scope of this work to study the performance of these molecular models with respect to their descriptive or predictive capabilities.

Most molecular simulation programs do not support point multipole sites, but only point charges $[8,9]$. Despite the fact that the point multipoles are computationally significantly less demanding than a corresponding arrangement of point charges [10], replacing point multipoles by a set of point charges can therefore be a necessity. From the physical perspective, it should be noted that multipoles are often better suited to describe the electrostatic molecular interactions [11]. Along these lines, Coen et al. [12] have studied point dipoles and analogous point charge sets for protein-protein interactions.

Because there is one degree of freedom for replacing point dipoles and point quadrupoles by point charges, this study was carried out to rationalize how this is best done and to give a recommendation for a point charge version of the multipolar models of Stoll et al. [1] and Vrabec et al. [2].

## 2. Replacing point multipoles by point charge arrangements

The symmetric two-centre LJ plus point dipole (2CLJD) or point quadrupole (2CLJQ) pair potentials are composed of two identical LJ sites a distance $L$ apart plus a point dipole of moment $\mu$ or a point quadrupole of moment $Q$ that is placed in the geometric centre of the molecular model and is aligned in the direction of the axis A connecting the two LJ sites. These point multipoles can be replaced by a close arrangement of two point charges $(+q,-q)$ in case of a point dipole or three point charges $(+q,-2 q,+q)$ in the case of a point quadrupole.

### 2.1. Replacement of a point quadrupole by three point charges

The charge magnitude $q$ in the arrangement $(+q,-2 q,+q)$ can be either negative or positive, depending on the sign of the quadrupole moment $Q$. To mimic the point quadrupole of a 2 CLJQ model, the central charge has to be placed in the geometric centre of the model and the two other charges on the axis A in a distance $\pm d$ from the centre. Note that the sign of the quadrupole moment does not play a role for pure component properties as studied in the present work, but it generally does in mixtures [5]. This charge distribution has a quadrupole moment [13, 14]

$$
\begin{equation*}
Q=2 q d^{2} \tag{1}
\end{equation*}
$$

and the octupole moment is zero. For replacing the point quadrupole by that type of point charge arrangement, either one of the parameters $d$ or $q$ can be specified and Equation (1) has to be used to determine the other parameter for the given value of $Q$. That choice is less trivial then it might seem. E.g. Ungerer et al. [15] have distributed the three charges on the molecular axis at $\pm 0.5 \mathrm{~A}$ from the centre of the $\mathrm{CO}_{2}$ model by Vrabec et al. [2], i.e. they used $d / \sigma \approx 1 / 6$, and obtained unfavourable VLE results.

In this work, the influence of the choice of $d$ on the saturated liquid density, vapour pressure and heat of vaporisation was investigated in detail for two selected substances, namely Chlorine $\left(\mathrm{Cl}_{2}\right)$ and the refrigerant 1,2-Dichloro-1,1,2,2tetrafluoroethane $\left(\mathrm{CF}_{2} \mathrm{Cl}_{-} \mathrm{CF}_{2} \mathrm{Cl}\right)$. Both the quadrupole moment $Q$ and the elongation $L$ increase from $\mathrm{Cl}_{2}$ to $\mathrm{CF}_{2} \mathrm{Cl}-\mathrm{CF}_{2} \mathrm{Cl}$, cf. Table 1. In a first step, the temperature was set to $0.7 T_{c}$, where $T_{c}$ is the critical temperature of the substance, while $d$ was varied and the quadrupolar moment was kept constant. The results are
shown in Figures 1 and 2. For small values of $d$ (corresponding to large values of $q$ ), numerical problems in the evaluation of the force field during molecular simulation become apparent. These are due to the contributions of point charges with a very large magnitude that have an alternating sign. Numerically, this translates to subtractions of large numbers. On the other hand, for large values of $d$, the error from approximating the point quadrupole by a spatially elongated point charge arrangement becomes important. This is visualized in Figure 3, where an equipotential line for a point quadrupole and for an according point charge arrangement with $d / \sigma=1 / 3$ is shown. It can be seen that the two electrostatic models deviate significantly from each other for this separation. Thus, there must be an optimum for the choice of $d$, for which the replacement is best suited for representing VLE properties. From Figures 1 and 2 it can be seen that this optimum is in the range of $1 / 20 \leq d / \sigma \leq 1 / 15$.

For $\mathrm{Cl}_{2}$, the present simulations were done with two different molecular simulation codes to check for generality and reproducibility of the results. These codes were our own group's software ms2 [10] and the Gibbs Ensemble Monte-Carlo (GEMC) code by Errington [16]. The simulation details are given in the Appendix. As can be seen in Figure 1, if $d$ is too small, the results from the point charge model may differ strongly from those of the point quadrupole model, which is clearly visible from the results of the GEMC code. $m s 2$ proved to be numerically more robust. This observation was confirmed by additional simulations for distances $d / \sigma$ $=1 / 40,1 / 60,1 / 80$ and $1 / 150$ in the case of $\mathrm{CF}_{2} \mathrm{Cl}-\mathrm{CF}_{2} \mathrm{Cl}$, which is more suitable for this study because of its very large quadrupole moment $Q=11.456 \mathrm{DA}$. No significant increase was visible in the relative error. In Figure 2, only the data points for the separations $d=\sigma / 40$ and $\sigma / 60$ are plotted for clarity. On the other hand, for large values of $d$, also large errors occur, but they are similar for both codes, because they are due to the inappropriateness of the physical approximation of the point quadrupole by three point charges.

Figure 4 shows a comparison of the results from the point quadrupole model and two versions of the corresponding point charge model ( $d=\sigma / 15$ and $\sigma / 20$ ) for the VLE properties of $\mathrm{CFCl}_{2}-\mathrm{CF}_{3}$ in the temperature range $0.65<T / T_{c}<0.95$. The $\mathrm{CF}_{2} \mathrm{Cl}-\mathrm{CF}_{2} \mathrm{Cl}$ model is very sensitive to variations in $d$, because of its large quadrupole moment and the wide separation $L=0.8 \sigma$ between the LJ sites. Both point charge models are in very good agreement with the original model. All model versions agree better with each other for higher temperatures, due to the fact that the effect of the quadrupole-quadrupole interaction decreases inversely proportional to $k_{B} T[14,17]$. This result is expected to be valid also in the case of the point dipole replacement, because the dipole-dipole interaction has the same proportionality $1 /\left(k_{B} T\right)[14,17]$ as discussed below.

To study whether these results can be generalized, all 21 quadrupolar models of Vrabec et al. [2] were simulated here. Again, two versions of the point charge model were investigated: $d=\sigma / 15$ and $\sigma / 20$. The simulations were carried out with $m s 2$ only.

For each substance, the VLE results based on two versions of the point charge models were compared to those from the original point quadrupole models of Vrabec et al. [2]. The comparison is summarized in Figure 5, the model parameters are given in Table 1. Generally, slightly better results were obtained for $d=\sigma / 20$, which is therefore recommended. Note that this recommendation is not independent on the numerical robustness of the simulation code, e.g. Figure 1 indicates that $\sigma / 10 \leq d \leq \sigma / 15$ may be a better choice for the GEMC code [16] used here.

Figure 5 shows that the point charge arrangement with $d=\sigma / 20$ rarely deviates from the original point quadrupole model by more than $0.1 \%$ for the saturated
liquid density and $0.4 \%$ for the heat of vaporisation. These numbers are well within the range of typical deviations from experimental data. The corresponding number for the vapour pressure is $5 \%$ and, hence, corresponds to the typical deviation from experimental data for that more sensitive property.

### 2.2. Replacement of a point dipole by two point charges

The point dipole of 2CLJD models was replaced by two point charges $(+q,-q)$, which were positioned symmetrically on both sides of the geometric centre along the axis A, separated by the distance $d$. This charge distribution has a dipole moment [13, 14]

$$
\begin{equation*}
\mu=q d \tag{2}
\end{equation*}
$$

and the quadrupole moment is zero. Again, the two parameters $q$ and $d$ can vary to specify a given value of the dipole moment $\mu$. The equipotential lines for a point dipole and a point charge representation with the same dipole moment and a separation of $d=\sigma / 3$ is shown in Figure 6.

As in the study above, three dipolar substances with medium to large dipole moments and small to large elongations $L$, namely 1,1-Difluoroethylene $\left(\mathrm{CF}_{2}=\mathrm{CH}_{2}\right)$, 1,1,1-Trifluoro-2-bromo-2-chloroethane $\left(\mathrm{CF}_{3}-\mathrm{CHClBr}\right)$ and Methylchloride $\left(\mathrm{CH}_{3} \mathrm{Cl}\right)$, were selected to investigate the dependence of the VLE properties on the separation $d$. Figures 7 to 9 show that the optimum distance is again in the range of $\sigma / 20 \leq d \leq \sigma / 15$. The numerical errors for short distances are not as dominant as for the point quadrupole, because the value of the charge magnitude $q$ increases only linearly with the decreasing distance $d$ and not quadratic as for the point quadrupole, cf. Equations (1) and (2) and Tables 1 and 2.

These findings were applied to 38 2CLJD models by Stoll et al. [1] for the recommended distance $d=\sigma / 20$. For each substance, the VLE results based on the point charge model were compared to those from the original point dipole model of Stoll et al. [1]. This comparison is summarized in Figure 10, the model parameters are given in Table 2. Figure 10 shows that the point charge arrangement with $d=\sigma / 20$ rarely deviates from the original point dipole model by more than $0.3 \%$ for the saturated liquid density and for the heat of vaporisation. The corresponding number for the vapour pressure is $5 \%$.

## 3. Conclusions

The substitution of point multipoles of 2CLJD and 2CLJQ potentials by linear point charge arrangements was studied. For both the point dipole and the point quadrupole, a separation of $d=\sigma / 20$ between the point charges is recommended to maintain the quality of the original models with respect to saturated liquid density, vapour pressure and heat of vaporisation. For the recommended distance of $d=\sigma / 20$, the deviations from the original models of the mentioned thermodynamic VLE properties were analysed for 38 dipolar and 21 quadrupolar models and the parameters for the corresponding point charge models were given. The deviations for saturated liquid density and heat of vaporisation rarely exceeded $0.3 \%$ and $0.4 \%$, respectively. The corresponding number for the vapour pressure is $5 \%$ and is thus in the range of typical deviations from experimental data for this sensitive property.

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## References

[1] J. Stoll, J. Vrabec and H. Hasse, J. Chem. Phys. 119, 11396 (2003).
[2] J. Vrabec, J. Stoll and H. Hasse, J. Phys. Chem. B 105, 12126 (2001).
[3]D. Möller and J. Fischer, Fluid Phase Equilib. 100, 35 (1994).
[4]G. A. Fernandez, J. Vrabec and H. Hasse, Int. J. Thermophys. 26, 1389 (2005).
[5] J. Vrabec, J. Stoll and H. Hasse, Mol. Sim. 31, 215 (2005).
[6] J. Stoll, J. Vrabec and H. Hasse, AIChE J. 49, 2187 (2003).
[7]J. Vrabec, G. K. Kedia and H. Hasse, Cryogenics 45, 253 (2005).
[8]B. Hess, C. Kutzner, D. van der Spoel and E. Lindahl, J. Chem. Theory Comput. 4, 435 (2008).
[9] Towhee monte carlo molecular simulation code, http://towhee.sourceforge.net.
[10] S. Deublein, B. Eckl, J. Stoll, S. V. Lishchuk, G. Guevara, C. W. Glass, T. Merker, M. Bernreuther, J. Vrabec and H. Hasse, Comp. Phys Commun. (2011), in press, doi:10.1016/j.cpc.2011.04.026.
[11] A. J. Stone, Science 321, 787 (2008).
[12]C. J. Coen, J. Newman, H. W. Blanch and J. M. Prausnitz, J. Coll. Int. Sci. 177, 276 (1996).
[13] W. Nolting, Grundkurs Theoretische Physik 3: Elektrodynamik (Springer-Verlag, Berlin, Heidelberg, 2007).
[14] J. M. Prausnitz, R. N. Lichtenthaler and E. G. de Azevedo, Molecular Thermodynamics of Fluid-Phase Equilibria, p. 60 (Prentice-Hall, Englewood Cliffs, New Jersey, 1999).
15]P. Ungerer, A. Wender, G. Demoulin, E. Bourasseau and P. Mougin, Mol. Sim. 30, 631 (2004).
[16] Errington monte carlo molecular simulation code, http://kea.princeton.edu/jerring/gibbs.
17] J. O. Hirschfelder, C. F. Curtiss and R. B. Bird, Molecular theory of gases and liquids, p. 835 (John Wiley and Sons, New York, London, Sydney, 1954).
[18] J. Vrabec and H. Hasse, Mol. Phys. 100, 3375 (2002).
[19] M. P. Allen and D. J. Tildesley, Computer simulations of liquids, p. 162 (Clarendon Press, Oxford, 1987).
[20] A. Z. Panagiotopoulos, Mol. Phys. 61, 813 (1987).
[21] J. Stoll, Molecular Models for the Prediction of Thermophysical Properties of Pure Fluids and Mixtures, Reihe 3, Nr. 836 (VDI-Verlag, Düsseldorf, 2005).

## Appendix A: Simulation details

The VLE calculations with $m s 2$ were done employing the Grand Equilibrium method [18]. The code structure of $m s 2$ is described in [10], as a compiler Intel ifort 11.1 was used and the simulations were executed on a cluster of Intel Xeon E5560 processors. The fluid was equilibrated over up to $26 \cdot 10^{6} \mathrm{MC}$ steps using 864 particles and the production run went up to $172 \cdot 10^{6} \mathrm{MC}$ steps. The electrostatic contributions were calculated by the reaction field method [19] with tinfoil boundary conditions. The cut-off radius was chosen to be $5 \sigma$ for all models with few exceptions and long-range corrections for energy and pressure were applied. If $5 \sigma$ exceeded half of the simulation box length, the value of the cut-off radius was adopted accordingly. The results for $\mathrm{Cl}_{2}, \mathrm{CF}_{2}=\mathrm{CH}_{2}$ and $\mathrm{CF}_{3}-\mathrm{CHClBr}$ were checked by a GEMC code [16, 20], where the total number of particles in both volumes was 1000 . The systems were equilibrated over up to $15 \cdot 10^{6} \mathrm{MC}$ steps and the number of production steps was up to $80 \cdot 10^{6}$. The electrostatic long range contributions were calculated in this case with the Ewald summation method [19]. The cut-off radius for the LJ interactions corresponded to half of the respective box length and long-range corrections for energy and pressure were applied.

Table 1. Molecular parameters of 2CLJQ type models for 21 substances. The quadrupole was modelled either by a point quadrupole with the moment $Q$ (numbers were taken from Vrabec et al. [2], except for $\mathrm{CF}_{2} \mathrm{Cl}-\mathrm{CF}_{2} \mathrm{Cl}$ where the numbers were taken from Stoll [21]) or as a corresponding point charge arrangement with $d=\sigma / 20$.

| \# | fluid | $\left(\epsilon / k_{B}\right) / \mathrm{K}$ | $\sigma / \mathrm{A}$ | $L / \sigma$ | $Q / \mathrm{DA}$ | $q / \mathrm{e}$ | $d / \mathrm{A}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{F}_{2}$ | 52.147 | 2.8258 | 0.5000 | 0.8920 | 4.652 | 0.14129 |
| 2 | $\mathrm{Cl}_{2}$ | 160.86 | 3.4016 | 0.5826 | 4.2356 | 15.242 | 0.17008 |
| 3 | $\mathrm{Br}_{2}$ | 236.76 | 3.5546 | 0.6126 | 4.8954 | 16.136 | 0.17773 |
| 4 | $\mathrm{I}_{2}$ | 371.47 | 3.7200 | 0.7200 | 5.6556 | 17.021 | 0.18600 |
| 5 | $\mathrm{N}_{2}$ | 34.897 | 3.3211 | 0.3151 | -1.4397 | -5.435 | 0.16606 |
| 6 | $\mathrm{O}_{2}$ | 43.183 | 3.1062 | 0.3123 | -0.8081 | -3.488 | 0.15531 |
| 7 | $\mathrm{CO}_{2}$ | 133.22 | 2.9847 | 0.8100 | -3.7938 | -17.736 | 0.14924 |
| 8 | $\mathrm{CS}_{2}$ | 257.68 | 3.6140 | 0.7418 | 3.8997 | 12.435 | 0.18070 |
| 9 | $\mathrm{C}_{2} \mathrm{H}_{6}$ | 136.99 | 3.4896 | 0.6809 | -0.8277 | -2.831 | 0.17448 |
| 10 | $\mathrm{C}_{2} \mathrm{H}_{4}$ | 76.950 | 3.7607 | 0.3376 | 4.3310 | 12.754 | 0.18804 |
| 11 | $\mathrm{C}_{2} \mathrm{H}_{2}$ | 79.890 | 3.5742 | 0.3637 | 5.0730 | 16.539 | 0.17871 |
| 12 | $\mathrm{C}_{2} \mathrm{~F}_{6}$ | 110.19 | 4.1282 | 0.6600 | -8.4943 | -20.759 | 0.20641 |
| 13 | $\mathrm{C}_{2} \mathrm{~F}_{4}$ | 106.32 | 3.8611 | 0.5800 | -7.0332 | -19.648 | 0.19306 |
| 14 | $\mathrm{C}_{2} \mathrm{Cl}_{4}$ | 211.11 | 4.6758 | 0.5672 | -16.143 | -30.751 | 0.23379 |
| 15 | propadiene | 170.52 | 3.6367 | 0.6863 | 5.1637 | 16.261 | 0.18184 |
| 16 | propyne | 186.43 | 3.5460 | 0.8000 | -5.7548 | -19.061 | 0.17730 |
| 17 | $\mathrm{SF}_{6}$ | 118.96 | 3.9615 | 0.6658 | 8.0066 | 21.248 | 0.19808 |
| 18 | $\mathrm{CF}_{4}$ | 59.235 | 3.8812 | 0.3582 | 5.1763 | 14.311 | 0.19406 |
| 19 | $\mathrm{CCl}_{4}$ | 142.14 | 4.8471 | 0.3496 | 14.346 | 25.431 | 0.24236 |
| 20 | propylene | 150.78 | 3.8169 | 0.6553 | 5.9387 | 16.977 | 0.19085 |
| 21 | $\mathrm{CF}_{2} \mathrm{Cl}-\mathrm{CF}_{2} \mathrm{Cl}$ | 183.26 | 4.3772 | 0.8000 | 11.456 | 24.902 | 0.21886 |

Table 2. Molecular parameters of 2CLJD type models for 38 substances. The dipole was modelled either by a point dipole with the moment $\mu$ (numbers were taken from Stoll et al. [1]) or as a corresponding point charge arrangement with $d=\sigma / 20$.

| \# | fluid | $\left(\epsilon / k_{B}\right) / \mathrm{K}$ | $\sigma / \mathrm{A}$ | $L / \sigma$ | $\mu / \mathrm{D}$ | $q / \mathrm{e}$ | $d / \mathrm{A}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | CO | 36.897 | 3.3009 | 0.3455 | 0.7378 | 0.9308 | 0.08253 |
| 2 | R11 | 224.15 | 4.0213 | 0.83 | 2.7009 | 2.7977 | 0.10051 |
| 3 | R12 | 185.66 | 3.8286 | 0.8541 | 2.3219 | 2.5293 | 0.09557 |
| 4 | R13 | 145.95 | 3.6184 | 0.8495 | 1.8261 | 2.1016 | 0.09046 |
| 5 | R13B1 | 170.32 | 3.6817 | 0.9119 | 2.0478 | 2.3162 | 0.09205 |
| 6 | R22 | 177.43 | 3.4682 | 0.8997 | 2.2667 | 2.7216 | 0.08671 |
| 7 | R23 | 123.56 | 3.2643 | 0.7864 | 2.1607 | 2.7563 | 0.08161 |
| 8 | R41 | 137.64 | 3.0382 | 0.8074 | 1.8850 | 2.5835 | 0.07596 |
| 9 | R123 | 221.75 | 4.0530 | 0.9130 | 3.7002 | 3.8016 | 0.10133 |
| 10 | R124 | 192.25 | 3.8852 | 1 | 3.2190 | 3.4502 | 0.09713 |
| 11 | R141b | 231.43 | 4.0209 | 0.8957 | 3.1484 | 3.2604 | 0.10053 |
| 12 | R142b | 193.68 | 3.8404 | 0.9029 | 2.9610 | 3.2107 | 0.09601 |
| 13 | R143a | 165.04 | 3.5960 | 0.9843 | 2.7470 | 3.1811 | 0.08990 |
| 14 | R152a | 182.01 | 3.5168 | 0.9419 | 2.7354 | 3.2390 | 0.08792 |
| 15 | $\mathrm{CH}_{3} \mathrm{Cl}$ | 186.57 | 3.3409 | 0.7700 | 2.0217 | 2.5200 | 0.08352 |
| 16 | $\mathrm{CH}_{3} \mathrm{Br}$ | 213.81 | 3.4557 | 0.7566 | 1.8536 | 2.2337 | 0.08639 |
| 17 | $\mathrm{CH}_{3} \mathrm{I}$ | 232.86 | 3.6367 | 0.7447 | 2.4983 | 2.8607 | 0.09092 |
| 18 | $\mathrm{CH}_{2} \mathrm{BrCl}$ | 274.49 | 3.5838 | 0.9951 | 3.1998 | 3.7179 | 0.08960 |
| 19 | $\mathrm{CHCl}_{3}$ | 265.29 | 3.8153 | 0.9907 | 3.3920 | 3.7024 | 0.09538 |
| 20 | $\mathrm{CHBr}_{3}$ | 357.41 | 4.0575 | 0.9716 | 3.5204 | 3.6130 | 0.10144 |
| 21 | $\mathrm{CHFCl}_{2}$ | 220.69 | 3.6522 | 0.9418 | 2.7852 | 3.1755 | 0.09131 |
| 22 | $\mathrm{CBrClF}_{2}$ | 212.23 | 3.8560 | 0.9325 | 2.6786 | 2.8927 | 0.09640 |
| 23 | $\mathrm{CBrCl}_{3}$ | 305.34 | 4.1366 | 0.9638 | 3.6313 | 3.6554 | 0.10342 |
| 24 | $\mathrm{CH}_{2} \mathrm{~F}-\mathrm{CH}_{3}$ | 176.84 | 3.3968 | 0.9128 | 2.4110 | 2.9557 | 0.08492 |
| 25 | $\mathrm{CHCl}_{2}-\mathrm{CH}_{2} \mathrm{Cl}$ | 286.36 | 4.0768 | 0.9835 | 4.2974 | 4.3896 | 0.10192 |
| 26 | $\mathrm{CHCl}_{2}-\mathrm{CH}_{3}$ | 255.24 | 3.8579 | 0.9885 | 3.5236 | 3.8033 | 0.09645 |
| 27 | $\mathrm{CCl}_{3}-\mathrm{CH}_{3}$ | 253.75 | 4.2224 | 0.8265 | 3.5019 | 3.4537 | 0.10556 |
| 28 | $\mathrm{CH}_{2} \mathrm{Cl}-\mathrm{CCl}_{3}$ | 292.86 | 4.3282 | 0.9533 | 4.7919 | 4.6102 | 0.10821 |
| 29 | $\mathrm{CH}_{2} \mathrm{~F}-\mathrm{CCl}_{3}$ | 259.97 | 4.1262 | 0.9323 | 4.3049 | 4.3444 | 0.10316 |
| 30 | $\mathrm{CF}_{3}-\mathrm{CHClBr}$ | 151.78 | 4.6727 | 0.4430 | 3.7380 | 3.3312 | 0.11682 |
| 31 | $\mathrm{CCl}_{3}-\mathrm{CF}_{2} \mathrm{Cl}$ | 258.25 | 4.3651 | 0.9746 | 4.7132 | 4.4964 | 0.10913 |
| 32 | $\mathrm{CHF}=\mathrm{CH}_{2}$ | 155.74 | 3.3552 | 0.8200 | 1.6565 | 2.0560 | 0.08388 |
| 33 | $\mathrm{CHCl}=\mathrm{CH}_{2}$ | 181.16 | 3.6875 | 0.6793 | 2.1078 | 2.3803 | 0.09219 |
| 34 | $\mathrm{CHCl}=\mathrm{CF}_{2}$ | 193.24 | 3.6501 | 0.9451 | 2.7449 | 3.1317 | 0.09125 |
| 35 | $\mathrm{CFCl}=\mathrm{CF}_{2}$ | 181.71 | 3.7438 | 0.9488 | 2.8408 | 3.1597 | 0.09360 |
| 36 | $\mathrm{CFBr}=\mathrm{CF}_{2}$ | 218.12 | 3.8290 | 0.9369 | 2.5273 | 2.7485 | 0.09573 |
| 37 | $\mathrm{CF}_{2}=\mathrm{CH}_{2}$ | 71.963 | 3.7848 | 0.3927 | 2.3643 | 2.6014 | 0.09462 |
| 38 | $\mathrm{CH}_{2} \mathrm{Br}-\mathrm{CH}_{3}$ | 255.75 | 3.6769 | 1 | 2.9425 | 3.3326 | 0.09192 |



Figure 1. Relative deviations of VLE properties between the point quadrupole (subscript PQ) and the corresponding point charge model for Chlorine $\left(\mathrm{Cl}_{2}\right)$ at $T / T_{c}=0.7$. The deviations are plotted over the separation $d$ between the point charges. Full symbols: GEMC code [16], empty symbols: $m s 2$ code [10]. Top: vapour pressure, centre: saturated liquid density, bottom: heat of vaporisation.


Figure 2. Relative deviations of VLE properties between the point quadrupole (subscript PQ) and the corresponding point charge model for 1,2 -Dichloro-1,1,2,2-tetrafluoroethane $\left(\mathrm{CF}_{2} \mathrm{Cl}-\mathrm{CF}_{2} \mathrm{Cl}\right)$ at $T / T_{c}=$ 0.7. The deviations are plotted over the separation $d$ between the point charges. The simulations were carried out with the $m s 2$ code [10]. Top: vapour pressure, centre: saturated liquid density, bottom: heat of vaporisation.


Figure 3. An equipotential line of a point quadrupole and a corresponding linear point charge arrangement with $d / \sigma=1 / 3$. Both electrostatic models are oriented along the $z$-axis, which is the spatial symmetry axis. Solid line: point quadrupole, dashed line: point charge arrangement.


Figure 4. VLE data for 1,2-Dichloro-1,1,2,2-tetrafluoroethane $\left(\mathrm{CF}_{2} \mathrm{Cl}^{2}-\mathrm{CF}_{2} \mathrm{Cl}\right)$. The figures show the results of the original model with a point quadrupole (open circle) compared to the results of corresponding models with point charges (full circle: $d / \sigma=1 / 20$, empty cross: $d / \sigma=1 / 15$ ): (a) vapour pressure, (b) saturated densities. The simulations were carried out with the $m s 2$ code [10].


Figure 5. Relative deviations of VLE properties between the point quadrupole model (subscript PQ) and the corresponding point charge model at $T / T_{c}=0.7$ for 21 fluids (cf. Table 1) and for two different point charge separations $d$. The simulations were carried out with the $m s 2$ code [10]. Full symbols: $d / \sigma=1 / 15$, empty symbols: $d / \sigma=1 / 20$. Note that the error bars accompanying the full symbols were omitted for clarity. Top: vapour pressure, centre: saturated liquid density, bottom: heat of vaporisation.


Figure 6. An equipotential line of a point dipole and a corresponding linear point charge arrangement with $d / \sigma=1 / 3$. Both electrostatic models are oriented along the $z$-axis, which is the spatial symmetry axis. Solid line: point dipole, dashed line: point charge arrangement.


Figure 7. Relative deviations of VLE properties between the point dipole model (subscript PD) and the corresponding point charge model for 1,1-Difluoroethylene $\left(\mathrm{CF}_{2}=\mathrm{CH}_{2}\right)$ at $T / T_{c}=0.7$. The deviations are plotted over the separation $d$ between the point charges. Full symbols: GEMC code [16], empty symbols: $m s 2$ code [10]. Top: vapour pressure, centre: saturated liquid density, bottom: heat of vaporisation.


Figure 8. Relative deviations of VLE properties between the point dipole model (subscript PD) and the corresponding point charge model for $1,1,1$-Trifluoro-2-bromo-2-chloroethane $\left(\mathrm{CF}_{3}-\mathrm{CHClBr}\right)$ at $T / T_{c}=$ 0.7. The deviations are plotted over the separation $d$ between the point charges. Full symbols: GEMC code [16], empty symbols: $m s 2$ code [10]. Top: vapour pressure, centre: saturated liquid density, bottom: heat of vaporisation.


Figure 9. Relative deviations of VLE properties between the point dipole model (subscript PD) and the corresponding point charge model for Methylchloride $\left(\mathrm{CH}_{3} \mathrm{Cl}\right)$ at $T / T_{c}=0.7$. The deviations are plotted over the separation $d$ between the point charges. The simulations were carried out with the $m s 2$ code [10]. Top: vapour pressure, centre: saturated liquid density, bottom: heat of vaporisation.


Figure 10. Relative deviations of VLE properties between the point dipole model (subscript PD) and the corresponding point charge model at $T / T_{c}=0.7$ for 38 fluids (cf. Table 2) and for a point charge separation of $d / \sigma=1 / 20$. The simulations were carried out with the $m s 2$ code [10]. Top: vapour pressure, centre: saturated liquid density, bottom: heat of vaporisation.


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